SET THEORY HOMEWORK 4

Due Thursday, March 30.

Problem 1. *Let M be a countable transitive model of ZFC and let $Add(\omega,1)$ be the poset of all partial finite functions from ω to $\{0,1\}$. Let G be a generic filter over M. As we did in class, define $f^*:\omega\to\{0,1\}$ to be $f^*=\bigcup G$. Recall that we proved in class that f^* is a total function with domain ω. Let $a=\{n\mid f^*(n)=0\}$.*

1. *Show that both a and $\omega\setminus a$ are unbounded subsets of ω.*
2. *Show that $a\notin M$.*

Problem 2. *Show that G is a $\mathbb{P}$-generic filter over M (i.e. a filter that meets every dense $D\subset\mathbb{P}$ in M) iff G is a filter such that G meets every maximal antichain $A\subset\mathbb{P}$ with $A\in M$.*

Problem 3. *Suppose that G is a $\mathbb{P}$-generic filter over M and $p\in G$.*

1. *Suppose that $D\subset\mathbb{P}$ is* **dense below** *p i.e. for every $q\leq p$, there is $r\leq q$ with $r\in D$. Show that $G\cap D\neq\emptyset$.*
2. *Let $A\subset\mathbb{P}$ be an antichain such that for every $q\in A, q\leq p$, and for every $r\leq p$, there is $q\in A$, such that r,q are compatible i.e. they have a common extension. Show that $G\cap A\neq\emptyset$. Such a set A is called a maximal antichain below p.*

Problem 4. *Let M be a countable transitive model of ZFC and $\mathbb{P}\in M$ be a poset. Suppose that σ and τ are two $\mathbb{P}$-names in M, such that both $\mathrm{dom}(\sigma),\mathrm{dom}(\tau)\subset\{\check{n}\mid n<\omega\}$. Let*

$$\pi=\{\langle\check{n},p\rangle\mid(\exists q,r)(p\leq q\wedge p\leq r\wedge\langle\check{n},q\rangle\in\sigma\wedge\langle\check{n},r\rangle\in\tau)\}.$$

Show that $\pi_G=\tau_G\cap\sigma_G$ for any generic filter G over M

Problem 5. *Let M be a countable transitive model of ZFC and $\mathbb{P}\in M$ be a poset. Suppose that σ is a $\mathbb{P}$-names in M, such that both $\mathrm{dom}(\sigma)\subset\{\check{n}\mid n<\omega\}$. Let*

$$\pi=\{\langle\check{n},p\rangle\mid(\forall q\in\mathbb{P})(\langle\check{n},q\rangle\in\sigma\to q\perp p)\}.$$

Show that $\pi_G=\omega\setminus\sigma_G$ for any generic filter G over M Hint: show that $\{r\mid\exists p\geq r(\langle\check{n},p\rangle\in\pi\vee\langle\check{n},p\rangle\in\sigma)\}$ is dense.